1. QUADRATIC EXPRESSIONS – Quick Review

1. If a, b, c are complex numbers and $a \neq 0$, then the expression $ax^2 + bx + c$ is called a ***quadratic expression***.
2. If a, b, c are complex numbers and $a \neq 0$, then $ax^2 + bx + c = 0$ is called a ***quadratic equation.***
3. A complex number α is said to be a ***root*** or ***solution*** of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.
4. The roots of $ax^2 + bx + c = 0$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
5. If α, β are the roots of $ax^2 + bx + c = 0$, then $\alpha + \beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$.
6. If α and β are the roots of $ax^2 + bx + c = 0$, then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$.
7. The quadratic equation having roots α, β is $(x - \alpha)(x - \beta) = 0$ or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
8. If $f(x) = ax^2 + bx + c =0$ is a quadratic equation then the quadratic equation whose roots are the reciprocals of the roots of $f(x) = 0$ is $f(1/x) = 0$.
9. If $f(x) = ax^2 + bx + c = 0$ is a quadratic equation then the quadratic equation whose roots are greater by k than those of $f(x) = 0$ is $f(x - k) = 0$.
10. If $f(x) = ax^2 + bx + c = 0$ is a quadratic equation then the quadratic equation whose roots are smaller by k than those of $f(x) = 0$ is $f(x + k) = 0$.
11. If $f(x) = ax^2 + bx + c = 0$ is a quadratic equation then the quadratic equation whose roots are numerically equal but opposite in ssign of the roots of $f(x) = 0$ is $f(-x) = 0$.
12. If $f(x) = ax^2 + bx + c = 0$ is a quadratic equation then the quadratic equation whose roots are multiplied by k of those of $f(x) = 0$ is $f(x/k) = 0$.
13. If $f(x) = ax^2 + bx + c = 0$ is a quadratic equation, then $b^2 - 4ac$ is called the ***discriminant*** of $ax^2 + bx + c = 0$.
14. If a, b, c are real then the nature of the rots of $ax^2 + bx + c = 0$ is as follows
    i) If $b^2 - 4ac < 0$, then the rots are imaginary. Further the roots are conjugate complex numbers.
    ii) If $b^2 - 4ac = 0$, then the roots are real and equal.
    iii) If $b^2 - 4ac > 0$, then the roots are real and not equal.
15. If a, b, c are rational then the nature of the roots of $ax^2 + bx + c = 0$ is as follows
    i) If $b^2 - 4ac < 0$, then the roots are imaginary. Further the roots are conjugate complex numbers.
    ii) If $b^2 - 4ac = 0$, then the roots are rational and equal.
    iii) If $b^2 - 4ac > 0$ and $b^2 - 4ac$ is a perfect square, then the roots are rational and not equal.
    iv) If $b^2 - 4ac > 0$ and $b^2 - 4ac$ is not a perfect square, then the roots are irrational and not equal. Further the roots are conjugate surds.
16. A necessary and sufficient condition for the quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ to have a common root is $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$. The common root is $\dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$.
17. If the roots of $ax^2 + bx + c = 0$ are imaginary (complex roots) then for $x \in R$, $ax^2 + bx + c$ and a have the same sign.
18. If the roots of $ax^2 + bx + c = 0$ are real and equal to $\alpha = -b/2a$, then for $\alpha \neq x \in R$, $ax^2 + bx + c$ and a have the same sign.

19. Let α, β be the real roots of $ax^2 + bx + c = 0$ and $\alpha<\beta$. Then
    i) $x \in R, \alpha < x < \beta \Rightarrow ax^2 + bx + c$ and a have opposite signs.
    ii) $x \in R, x < \alpha$ or $x > \beta \Rightarrow ax^2 + bx + c$ and a have the same sign.
20. For $x \in R$, the sign of a quadratic expression $ax^2 + bx + c$ is same as that of 'a' except when the roots of the equation $ax^2 + bx + c = 0$ are real and x lies between them.
21. Let $f(x) = ax^2 + bx + c$ be a quadratic function.
    i) If $a > 0$ then f(x) has minimum value at $x = \frac{-b}{2a}$ and the minimum value = $\frac{4ac - b^2}{4a}$.
    ii) If $a < 0$ then f(x) has maximum value at $x = \frac{-b}{2a}$ and the maximum value $\frac{4ac - b^2}{4a}$.
22. If $ax^2 + bx + c$ is a quadratic expression, then $ax^2 + bx + c > 0$ or $ax^2 + bx + c \geq 0$ or $ax^2 + bx + c < 0$ or $ax^2 + bx + c \leq 0$ is called a ***quadratic inequation*** or ***quadratic inequality.***
23. If a, b, c, f, g, h are complex numbers and at least one of a, h, b is nonzero, then $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ is called a ***quadratic expression*** or ***second degree linear expression*** in x and y.
24. If a, h, b are complex numbers and atleast one of them is nonzero, then $ax^2 + 2hxy + by^2$ is called a ***second degree homogeneous expression*** in x and y.
25. Every second degree homogeneous expression in x and y can be resolved into two linear factors.
26. The necessary and sufficient condition that $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ can be resolved into two linear factors is $\Delta=abc+2fgh-af^2 - bg^2 - ch^2=0$.
27. S can be resolved into two real linear factors $\Leftrightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$. $h^2 \geq ab$, $g^2 \geq ac$, $f^2 \geq bc$.
28. The condition that the roots of $ax^2 +bx + c = 0$ may be in the ratio m : n is $mnb^2= ac (m + n)^2$.
29. If a + ib is a root of $px^2 + qx + r = 0$, p, q, r $\in$ R, then the other root is a – ib.
30. If $a \pm \sqrt{b}$ is a root of $px^2 + qx + r = 0$, then the other root is $a \mp \sqrt{b}$.(p, q, r $\in$ Q)
31. If x > 0 then the least value of x + 1/x is 2 and if x < 0, x + 1/x < –2.
32. The condition that the roots of $ax^2 + bx + c =0$ are in the ratio m : n is $mn b^2 = (m +n)^2 ac$
33. The condition that one root of $ax^2 + bx + c =0$ may be k times the other root is $(k + 1)^2+ ac = kb^2$
34. If one root of the quadratic equation $ax^2 + bx + c =0$ is equal to the nth power of the other then $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$
35. The condition that one root of $ax^2 + bx + c= 0$ may be the square of the other is
    $ac (a + c) + b^3 = 3abc$
36. If the ratio of the roots of the equation $ax^2 + bx + c = 0$ is same as three ratio of the roots of $px^2 + qx + r=0$. Then $\frac{b^2}{ac} = \frac{q^2}{pr}$.
37. If the roots of the equation $ax^2 + bx + c=0$ be the square roots of the roots of the equation $px^2 + qx + r = 0$. Then $2apc = pb^2 + qa^2$.
38. If a + b + c= 0. Then the roots of $ax^2 + bx + c= 0$ are 1, c/a.
39. If a + c = b. Then the roots of $ax^2 + bx + c= 0$ are –1, –c/a.
40. The condition that the roots of the equation $ax^2 + bx + c = 0$ are reciprocal of those of $px^2 + qx + r = 0$ is $acq^2 = b^2 pr$.
41. If α, β are the roots of $f(x) = ax^2 + bx + c =0$. Then the equation having roots

i) $\frac{1-\alpha}{1+\alpha}, \frac{1-\alpha}{1+\alpha}$ is $f\left(\frac{1-x}{1+x}\right)=0$

ii) $\frac{\alpha+p}{q}, \frac{\beta+p}{q}$ is f(xq – p) = 0

iii) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ is f $\left(\left(\frac{x}{1-x}\right)=0\right.$

42. $\frac{p-\alpha}{q+\alpha}, \frac{p-\beta}{q+\beta}$ is $f\left(\frac{p-qx}{1+x}\right)=0$

43. If x > 0. Then the least of x + 1/x is 2.

44. If x > 0. Then the least value of ax + $\frac{b}{x}$ is $2\sqrt{ab}$.

45. If a_1, a_2……a_n are positive, then the least value of $(a_1 + a_2 + \ldots\ldots a_n)\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots\ldots\frac{1}{a_n}\right)$ is n^2.

46. If a > 0. Then $\sqrt{a+\sqrt{a+\sqrt{a+\ldots\ldots\infty}}} = \frac{1+\sqrt{4a+1}}{2}$.